Math 1111 Challenge-for-Credit Sample Exam

Name: ______________________________

MtA ID: _____________________________

Instructions: Read all questions before attempting. Marks for each question are indicated in square brackets. There are 75 marks total. No marks will be awarded for illegible work. Non-graphing calculators are allowed, but no other aids are permitted. You may use the backs of pages for rough work. You have three hours. Good luck!

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1. Evaluate the following limits. If using l'Hopital's rule, you must provide justification.

   (a) [2] $\lim_{x\to 1} \dfrac{x^2 - x}{x^2 - 3x + 2}$

   (b) [2] $\lim_{x\to 3} \dfrac{\frac{1}{x+2} - \frac{1}{5}}{x - 3}$

   (c) [2] $\lim_{x\to 0} \dfrac{2x + \cos(x)}{3x + \sin(x) + 1}$

2. [3] Let $f(x) = \begin{cases} x+4 & x<2 \\ 6 & x=2 \\ x^2-1 & x>2 \end{cases}$. Is f continuous at $x = 2$? Justify your answer.

3. [3] Find the equation of the tangent to the curve $y = x^3 - 3$ at the point $(2, 5)$.

4. [3] Find the function f so that $f'(x) = \frac{3}{x} + 5$ and $f(1) = 4$.

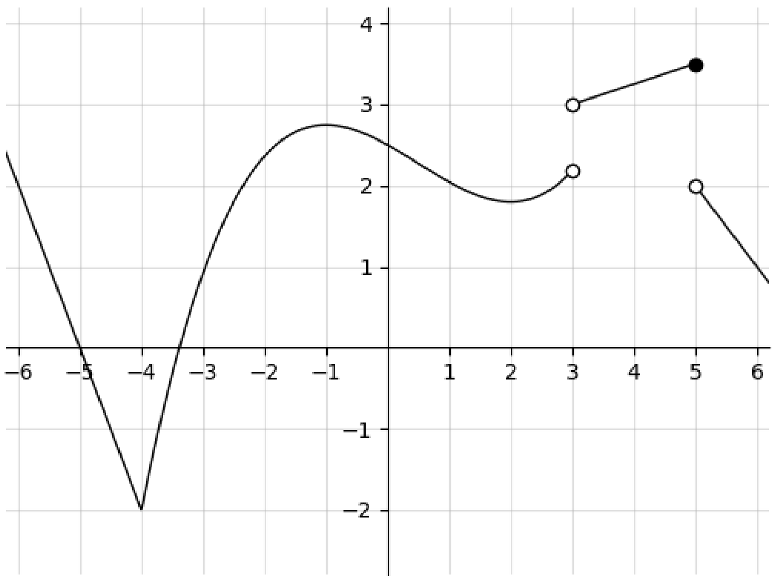
5. For the function f whose graph is given below, find:

(a) [1] The value(s) of x for which f is not continuous: ________________

(b) [1] The value(s) of x for which f' is not defined: ________________

(c) [1] The value(s) of x for which f' is equal to zero: ________________

(d) [1] The value(s) of x for which f has a critical point: ________________



6. [4] Sketch a rough graph of a single function $f(x)$ which has the following features:

- $f(-2) = 3$
- $\lim_{x \to 4^-} f(x) = \infty$
- $\lim_{x \to 4^+} f(x) = -1$
- $\lim_{x \to -\infty} f(x) = -3$
- $f(5) = 0$

7. (a) [1] For a function $f(x)$, state the definition of $f'(a)$ (either form is acceptable).

(b) [4] If $f(x) = 3\sqrt{x}$, find $f'(4)$ using only the definition of the derivative.

8. [2] If $1 \leq f(x) \leq x^2 + 2x + 2$ for all x, find $\lim_{x \to -1} f(x)$.

9. [2] Explain why there is a solution to the equation $x^5 - x^3 + 3x - 5 = 0$ between $x = 1$ and $x = 2$.

10. [4] Show that the equation $1 + 2x + x^3 + 4x^5$ has at least one root and not more than one root.

11. [10] Find the derivatives of the following functions. You do not need to simplify your answer after taking the derivative. State the domain of each of the derivative functions.

(a) $f(x) = (x^2 + x^3)(x + 1)$

(b) $y = \ln(\frac{\pi}{3} \cos x)$

(c) $s(t) = 5 \arctan(t^3 + 1)$

(d) $y = \dfrac{1}{1 + \frac{1}{x}}$

12. Suppose that a virus is spreading amongst the students in a university residence. Initially, only 10 are infected, but everyday after that an additional 5% are infected.

 (a) [1] If $I(t)$ is the number of students infected t days after the initial 10 students are infected, give a formula for $I(t)$.

 (b) [2] Find the rate of increase of the number of students infected exactly 30 days after the initial 10 students are infected.

13. (a) [1] Find the linear approximation to the function $f(x) = \ln(x)$ at the point $a = 1$.

 (b) [1] Use the above to estimate the value of $\ln(0.9)$.

14. [3] Use implicit differentiation to find $\frac{dy}{dx}$ if $xy = \sin(x+y)$.

15. [5] A plane flying horizontally at an altitude of 1 km and a speed of 200 km/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when the plane is 2 km away from the station.

    When presenting your solution, be sure to declare any relevant variables, draw relevant diagrams, and completely explain your work.

16. [5] A farmer with 750 ft of fencing wants to enclose a rectangular area, and then divide that area into four equal pens, with the interior fencing parallel to at least one side of the original rectangle. What is the largest possible total area of the four pens?

    When presenting your solution, be sure to declare any relevant variables, draw relevant diagrams, and completely explain your work.

17. [3] Find all critical points of the function $g(x) = x^2 e^x$. For each critical point, identify whether it is a local minimum, a local maximum, or neither.

18. For the function $f(x) = 20 + 8x^3 + x^4$:

(a) [2] At what value(s) of x does f have a critical point? ____________________

(b) [1] On which interval(s) is f increasing? ____________________

(c) [2] On which interval(s) is f concave up? ____________________

(d) [3] Give a rough graph of the function which gives all local maximum or minimum points and inflection points.

(Use the space below to work out your answers)